

**BACCALAURÉAT GÉNÉRAL  
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES  
MATHÉMATIQUES – ANGLAIS**

**SUJET 12**

**Zeno's paradox  
Thème : sequences**

**Sujet comportant une page. L'usage de tout modèle de calculatrice, avec ou sans mode examen est autorisé.**

Zeno of Elea (c. 450 BC) is well known for creating the paradox of the Tortoise and Achilles (Achilles was the great Greek hero of Homer's *The Iliad*).

The Tortoise challenged Achilles to a race, and because Achilles was a mighty warrior whereas the Tortoise was heavy and slow, the tortoise was given a head start of 100 meters.

We suppose the tortoise speed is one tenth of Achilles'.

The race starts:

- 1<sup>st</sup> step: While Achilles runs 100 meters, the tortoise only moves 10 meters forward. Achilles is now 10 meters behind the tortoise.
- 2<sup>nd</sup> step: While Achilles runs 10 meters, the tortoise only moves 1 meter forward. Achilles is now 1 meter behind the tortoise.
- 3<sup>rd</sup> step: While Achilles runs 1 meter, the tortoise only moves 0.1 meter forward. Achilles is now 0.1 meter behind the tortoise.

And the race goes on. After a few more steps, Achilles realizes that he will never catch up the tortoise.

**Questions:**

1. Describe in a few words what the text deals with.
2. Why is there a paradox?

**Exercise:**

Let  $(u_n)$  be a sequence where  $u_n$  is the distance separating Achilles and the tortoise (in meters) after  $n$  steps.

1. Calculate  $u_0$ ,  $u_1$ ,  $u_2$  and  $u_3$
2. Express  $u_n$  in function of  $n$ . What is the nature of this sequence? What are its characteristics?
3. Calculate  $\lim_{n \rightarrow \infty} u_n$ .
4. How can we solve Zeno's paradox?

Le sujet doit être restitué à la fin de l'épreuve.