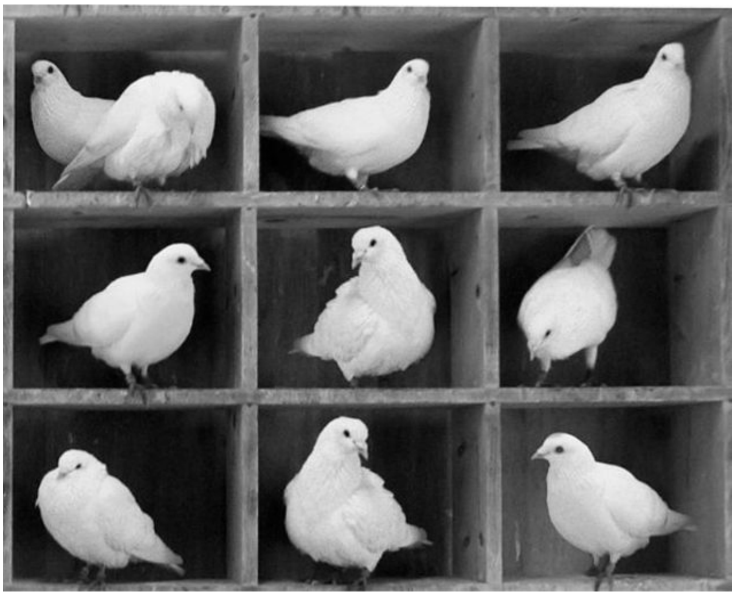
BACCALAURÉAT GÉNÉRAL
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 9

The Pigeonhole Principle
Reasoning

Ce sujet comporte deux pages. L'usage de tout modèle de calculatrice, avec ou sans mode examen, est autorisé.

Pigeonhole principle:



In mathematics, the pigeonhole principle states that if n items are put into m containers, with $n > m$, then at least one container must contain more than one item.

This theorem is exemplified in real life by truisms like "in any group of three gloves there must be at least two left gloves or two right gloves". It is an example of a counting argument. This seemingly obvious statement can be used to demonstrate possibly unexpected results; for example, that there are two people in London who have the same number of hairs on their heads.

The first formalization of the idea is believed to have been made by Peter Gustav Lejeune Dirichlet in 1834 under the name *Schubfachprinzip* (*drawer principle* or *shelf principle*). But he’s also credited with being one of the first mathematicians to give the modern formal definition of a function.

Source: Wikipedia, the free encyclopedia

Vous devez restituer le sujet à la fin de l’épreuve

I. Explain what the text deals with and comment on it.

II. Exercise.

1. Knowing that a human has a maximum of 150 000 hairs on his head, what information do you need to justify that, at least, two Londoners have the same number of hairs?

2. In a drawer, you have 6 black socks, 4 green socks and 10 white socks, all mixed. You randomly pick one sock, note its colour and pick another sock from the same drawer. How many socks must you randomly pick to be sure to get a matching pair?

3. You pick five numbers from the set of integers 1 to 8. We consider event E: “the sum of two of them is equal to 9”.

a. List all the couples of integers whose sum is nine.

b. Using the pigeonhole principle, what can you say of event E.

4. Prove that among any five points selected inside an equilateral triangle with all sides equal to 1, there always exists a pair whose distance is not greater than 0.5.