

**BACCALAURÉAT GÉNÉRAL  
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES  
MATHÉMATIQUES – ANGLAIS**

**SUJET 20**

**Magical Multiplying Method  
Sequences**

Ce sujet comporte deux pages. L'usage de tout modèle de calculatrice, avec ou sans mode examen, est autorisé.

I will remember the wonder that I experienced when I learned "**MULTIPLY by ADDING**", using a much simpler, arithmetic basis. Before we can proceed, we must introduce triangular numbers.

$$1 = \mathbf{1}$$

$$1+2 = \mathbf{3}$$

$$1+2+3 = \mathbf{6}$$

$$1+2+3+4 = \mathbf{10}$$

So, 1, 3, 6 and 10 are the first four triangular numbers. And with these few examples we can see a short cut for finding other triangular numbers.

We will call  $T_n$  the  $n$ th triangular number. So  $T_1 = 1$ ,  $T_2 = 3$ ,  $T_3 = 6$ ....

Finally we're ready to show you the magical way to multiply without multiplying anything. We call it the "Magical Multiplying Method" (or the "3M" way). Let's take as an example  $15 \times 9$ .

1. Take the larger factor 15 and find  $T_{15}$ .
2. Subtract 1 from 9, the smaller factor, getting 8. Find  $T_8$ .
3. Subtract the two factors,  $15 - 9$ ; that's 6. Find  $T_6$ .
4. Add the results of Steps 1 and 2, then subtract the result from Step 3. That's your product!

*Adapted from <http://www.trottermath.net/algebra/3mgame.html>*

**I. Explain what the texts deal with and comment on it.**

**II. Exercises:**

1. Without using the text, explain how to complete the following table:

$n$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
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Vous devez restituer le sujet à la fin de l'épreuve

$T_n$	1	3	6	10	15	21	28	36	45	...	...	...	...	...	...
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2. Check the magical multiplying method works with the example given in the text.
3. Use this method to calculate  $6 \times 14$ .
4. Explain why for each integer  $n > 0$ :  $T_{n+1} = T_n + n$ .
5. Assuming that  $T_n = \frac{n(n+1)}{2}$ , prove the “3M” method by showing that for any whole numbers  $a$  and  $b$  with  $a$  greater than  $b$ :  $T_a + T_{b-1} - T_{a-b} = a \times b$
6. What happens if  $a$  is equal to  $b$ ?